



LEARNING DIFFICULTIES IN SOLID GEOMETRY AMONG UNIVERSITY STUDENTS: THE ROLE OF VISUAL–SPATIAL SKILLS

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ABSTRACT

Solid geometry is an essential area of advanced mathematics that studies three-dimensional shapes like prisms, cones, cylinders, spheres, and polyhedra. In higher education, many students face serious learning problems in solid geometry because of the abstractness of ideas in three-dimensional space and the higher-level cognitive processes involved in visualizing geometric structures in Euclidean space. These learning problems often involve issues of mental rotation, incorrect interpretation of two-dimensional representations, and mechanical memorization of formulas for volume and surface area without understanding. This paper explores the theoretical basis of solid geometry by examining basic mathematical concepts such as coordinate points in three-dimensional space, distance formulas, and plane equations. It also highlights the importance of visual-spatial skills in geometry learning, especially in tasks involving spatial transformation, cross-section understanding, and perspective understanding. Key concepts, such as Euler's polyhedron formula and the three-dimensional form of the Pythagorean theorem, are introduced to demonstrate the logical organization of solid geometry concepts.

Keywords: Solid Geometry; Visual–Spatial Skills; Mental Rotation; Spatial Visualization; University Students; Learning Difficulties.

1. INTRODUCTION

Solid geometry is one of the most significant and complex branches of mathematics that studies three-dimensional shapes. It is the basis for many university fields of study, including engineering, architecture, robotics, physics, and computer graphics. In contrast to plane geometry, which studies two-dimensional shapes, solid geometry involves knowledge of shapes in Euclidean three-dimensional space, where points are defined by three spatial coordinates.

Euclidean three-dimensional space:

$$\mathbb{R}^3$$

Point representation:

$$P(x, y, z)$$

The concept of solids like cubes, prisms, pyramids, cylinders, cones, and spheres encompasses important mathematical concepts like volume and surface area. For instance, the volume of a prism is measured by the area of its base and its height. Likewise, the volume of a cylinder is measured by the radius of its circular base and its height. The volume of a sphere is measured by the formula involving the radius of the sphere. Though these formulas are used extensively in science and technology, their proper application requires robust conceptual and spatial skills.

Volume of a prism:

$$V = Bh$$

Volume of a cylinder:

$$V = \pi r^2 h$$

Volume of a sphere:

$$V = \frac{4}{3} \pi r^3$$

Research indicates that many university students experience problems with solid geometry, especially in visualizing three-dimensional figures from two-dimensional representations, understanding transformations in space, applying formulas correctly, and proving statements about polyhedra. These problems are inextricably tied to visual-spatial skills, which entail the capacity to mentally manipulate and rotate objects in space. For example, calculating the space diagonal of a cuboid involves three-dimensional spatial thinking that goes beyond algebraic calculation.

Space diagonal of a cuboid:

$$d = \sqrt{a^2 + b^2 + c^2}$$

Solid geometry also involves other complex relationships between polyhedra and their vertices, edges, and faces. One of the most important achievements in this field is Euler's formula, which may require high-level spatial and logical thinking. Thus, difficulties in learning solid geometry are not only mathematical but also cognitive, which is a reflection of the differences in the visual-spatial abilities of students. Such differences have a significant impact on learning geometry at a university level.

Euler's polyhedron formula:

$$V - E + F = 2.$$

2. LITERATURE REVIEW

Purnomo and Machromah (2018) investigated the key difficulties faced by university students in learning solid geometry, especially concentrating on the development of visual-spatial skills. The authors concluded that students often faced difficulties in comprehending three-dimensional objects when they were represented in two-dimensional formats, like illustrations in textbooks or diagrams. The authors pointed out that learners often faced difficulties in mental rotation, orientation, and geometric relationships in solids. The authors stressed that the learning of solid geometry demanded high visualization skills, and they concluded that the enhancement of visual-spatial skills in students could help overcome difficulties and improve performance in problem-solving at the university level.

Nagubandi (2018) conducted an investigation on the effects of visual-spatial difficulties on learning geometry. The study pointed out that student who had difficulties in spatial skills continued to struggle with understanding geometric figures, particularly in transformation, rotation, and spatial skills. Nagubandi concluded that students who lacked visual-spatial competence struggled with conceptual understanding and often produced misconceptions in learning geometry. The study concluded that geometry performance was significantly affected by students' mental manipulation of geometric shapes and visualizing spatial relationships, implying that training in spatial skills should be incorporated into geometry teaching.

Abd Wahab et al. (2016) conducted a case study investigation on the relationship between visual-spatial abilities and geometric thinking levels in learning three-dimensional geometry concepts among high-achieving students. The results of the study revealed that even high-achieving students experienced challenges in mastering advanced 3D geometry concepts if their visual-spatial abilities were not adequate. The study established that students' capabilities in visualizing geometric solids

mentally were directly related to their development in geometric thinking. The researchers concluded that visual-spatial skills played a crucial role in facilitating high-level geometric thinking and suggested that teachers should design learning experiences to enhance these cognitive skills.

3. THEORETICAL FOUNDATIONS OF SOLID GEOMETRY

Theoretical basis of solid geometry is the mathematical basis required for understanding three-dimensional geometric figures. In higher education, solid geometry is a vital continuation of classical Euclidean geometry, which goes beyond two-dimensional geometry to explore the properties of objects that exist in three-dimensional space. This area of geometry is vital for understanding the properties and relationships of solids like polyhedra, spheres, cylinders, and cones.

Theoretical concepts like Euclidean space, coordinate representation of points, calculation of distance, and equations of planes are the foundation for more complex concepts in spatial geometry. It is vital to have a proper understanding of these theoretical concepts not only for problem-solving in geometry but also for the development of visual-spatial skills necessary for success in advanced mathematics.

3.1 Definition of Solid Geometry

Solid geometry is the branch of geometry concerned with three-dimensional objects.

A solid figure is defined as:

$$S \subseteq \mathbb{R}^3$$

where $\mathbb{R}^3$ is three-dimensional Euclidean space.

3.2 Euclidean Space and Coordinates

A point in 3D space is represented as:

$$P(x, y, z)$$

The distance between two points:

$$P_1(x_1, y_1, z_1), P_2(x_2, y_2, z_2)$$

is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3.3 Planes in Solid Geometry

A plane in space is given by the equation:

$$Ax + By + Cz + D = 0$$

where (A, B, C) is the normal vector.

4. VISUAL-SPATIAL SKILLS IN GEOMETRY LEARNING

Visual-spatial skills are very important in the learning and understanding of geometry, especially in the area of solid geometry, where students have to think about objects in three-dimensional space. Unlike two-dimensional geometry, which can easily be depicted on a flat surface, solid geometry requires students to think about depth, hidden surfaces, and spatial relationships that are not immediately visible. Visual-spatial skills are thus very important in the interpretation of diagrams, solving spatial problems, and understanding complex geometric concepts. This section will discuss the definition of visual-spatial skills and its cognitive importance in the learning of solid geometry, especially its impact on students' understanding and problem-solving abilities at the university level.

4.1 Definition of Visual-Spatial Ability

Visual-spatial skills are basic cognitive abilities that entail the ability of an individual to perceive, interpret, and mentally manipulate objects and their relationships in three-dimensional space. In the context of learning geometry, visual-spatial skills enable a student to form correct mental images of objects, manipulate them mentally, and comprehend spatial relationships that cannot be easily perceived from two-dimensional images.

Visual-spatial skills are essential in mathematics because many geometric concepts require a learner to mentally visualize objects beyond what is visible on paper. University students, in particular, are required to interpret diagrams, mentally rotate solids, and comprehend spatial relationships such as the intersection of planes, projections, and cross-sections.

Point representation in Euclidean space:

$$P(x, y, z)$$

This coordinate system forms the basis for understanding the placement of objects and relationships between geometric structures.

4.2 Cognitive Role in Solid Geometry

The subject of solid geometry involves the construction of mental images of three-dimensional figures. Unlike plane geometry, where figures can be completely observed in two dimensions, solid geometry involves the ability to mentally visualize depth, hidden surfaces, and dimensions that cannot be directly depicted on flat paper.

For instance, comprehension of a cube involves mental visualization of:

- its six faces
- twelve edges
- eight vertices
- angles between intersecting planes
- hidden parts not visible in a diagram

Students with poor visual-spatial skills will have difficulty visualizing these relationships in their minds. This will cause them to not comprehend simple geometric concepts, even if they can commit formulas to memory.

One of the important cognitive difficulties is estimating distances and relationships in three-dimensional space. For example, the distance between two points is more than a two-dimensional formula.

Distance between two points in space:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equally, students need to grasp the three-dimensional extensions of classical results. A typical example is calculating the space diagonal of a cuboid, which involves visualizing a line passing through the interior of the solid.

Space diagonal of a cuboid:

$$d = \sqrt{a^2 + b^2 + c^2}$$

Such outcomes underscore the heavy reliance of solid geometry on visualization.

Another critical consideration is the capability for mental rotation, which is a crucial requirement for comprehending transformations like rotation and reflection in three-dimensional space. Such transformations can be expressed in a mathematical formulation through rotation matrices.

Rotation in three dimensions about the z-axis:

$$R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This means that spatial reasoning involves both cognitive visualization and mathematical structure. Moreover, understanding the cross-section of a solid involves visualizing how a plane intersects a three-dimensional object. The equation of a plane is basic in this visualization.

General equation of a plane:

$$Ax + By + Cz + D = 0$$

The understanding of the intersection of planes and solids can easily become problematic for students with weak spatial skills.

Thus, visual-spatial skills are not supplementary but rather crucial tools for learning solid geometry. Students with stronger spatial skills are likely to perform better in geometry problems related to proofs, diagrams, transformations, and applications, while students with weaker skills are likely to have problems with conceptual understanding and problem-solving in three-dimensional mathematics.

5. COMMON LEARNING DIFFICULTIES IN SOLID GEOMETRY

Solid geometry is a crucial part of mathematics in higher education institutions, but it is also one of the most difficult topics for university students, as it is highly dependent on mental spatial abilities and three-dimensional visualization. In contrast to plane geometry, where geometric figures can be visually inspected and analyzed on a plane surface, solid geometry requires learners to mentally create, manipulate, and interpret geometric figures that exist in three-dimensional space. Consequently, university students often face substantial learning difficulties when dealing with spatial transformations, analyzing geometric figures, and applying geometric formulas. These learning difficulties are directly related to visual-spatial skills, which play a crucial role in determining the ability of university students to comprehend geometric figures from a conceptual perspective rather than from memorization. This section will examine the most frequent learning difficulties that university students face in solid geometry, including difficulties with mental rotation, interpreting two-dimensional figures, and applying geometric formulas without a deeper understanding of spatial geometry.

5.1 Difficulty in Mental Rotation

Among the most frequent problems encountered by university students in solid geometry is the problem of mental rotation. Indeed, many students have problems conceptualizing the change that occurs in a three-dimensional figure when it is rotated or viewed from a different perspective. Problems involving the mental rotation of prisms, understanding the orientation of pyramids, or visualizing polyhedra from different angles are often confusing to students. This problem directly impacts the students' capacity to conceptualize fundamental geometric ideas like cross-sections, projections, and transformations. It is difficult for students to mentally manipulate solids and predict how they will look after being moved in space if they are not able to mentally rotate solids.

Mental rotation and spatial reasoning are very much related to geometric transformations that can be represented mathematically by rotation matrices.

Rotation of an object about the z-axis:

$$R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Mental rotation is also needed when visualizing diagonals inside solids, such as the space diagonal of a cuboid.

Space diagonal of a cuboid:

$$d = \sqrt{a^2 + b^2 + c^2}$$

5.2 Misinterpretation of Diagrams

Another major learning difficulty comes from the incorrect interpretation of geometric diagrams. Because most three-dimensional objects are represented two-dimensionally on paper or computer screens, learners are forced to use their spatial skills to visualize depth. For instance, a two-dimensional representation of a cube or cuboid may look confusing, as if parallel sides are intersecting and where depth is not explicitly shown. Consequently, learners may misinterpret the actual shape, position, or hidden faces of the three-dimensional object. This leads to conceptual misunderstandings when working mathematically with angles between planes, diagonals, or intersections of solids.

Knowledge of spatial planes is critical in interpreting cross-sections and intersections.

General equation of a plane:

$$Ax + By + Cz + D = 0$$

Students also face difficulty in calculating distances between points in three-dimensional diagrams.

Distance between two points in space:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Misinterpretation of diagrams often prevents learners from correctly identifying these spatial relationships.

5.3 Formula Memorization Without Understanding

Another problem in solid geometry is that students are prone to committing formulas for the volume and surface area to memory without building a conceptual understanding of the three-dimensional object. Many students are using formulas in problem-solving without conceptualizing how the three-dimensional object is created or why the formula is true. This causes problems when students are asked to make comparisons among objects, point out cross-sectional areas, or establish relationships among dimensions.

One such problem is the formula for the volume of a cylinder.

Volume of a cylinder:

$$V = \pi r^2 h$$

Similarly, students often memorize the volume of a cone without understanding its derivation from the cylinder.

Volume of a cone:

$$V = \frac{1}{3} \pi r^2 h$$

Another frequently memorized result is the volume of a sphere.

Volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

Students may also struggle with surface area formulas, especially when solids involve curved surfaces.

Surface area of a cylinder:

$$A = 2\pi rh + 2\pi r^2$$

Surface area of a sphere:

$$A = 4\pi r^2$$

Without understanding the geometric meaning behind such formulas, students often fail to connect mathematical computation with the real spatial properties of solids.

6. FUNDAMENTAL THEOREMS IN SOLID GEOMETRY

The foundation of solid geometry is based on a number of key theorems that express fundamental properties between the components of three-dimensional shapes. These theorems not only provide a basis for mathematical calculation methods, such as calculating distances, volumes, and surface areas, but also provide a means for advanced logical thinking. Some of the most important results include Euler's polyhedron formula, the three-dimensional form of the Pythagorean Theorem, and the general volume theorem for prisms. These theorems are essential for students to understand, as they provide the theoretical framework for advanced spatial problem-solving.

6.1 Euler's Polyhedron Formula

Theorem (Euler)

For any convex polyhedron, the relationship between its vertices, edges, and faces is given by:

$$V - E + F = 2$$

where:

- V represents the number of vertices
- E represents the number of edges

- F represents the number of faces

Proof (Sketch)

To understand Euler's formula, consider a convex polyhedron projected onto a plane. This projection forms a connected planar graph. In planar graph theory, the following relation holds:

$$V - E + F = 1 + C$$

where C is the number of connected components. Since a convex polyhedron forms a single connected structure, we have:

$$C = 1$$

Substituting this value gives:

$$V - E + F = 2$$

Thus, Euler's formula is valid for all convex polyhedra.

Example: Cube

For a cube:

- Vertices: $V = 8$
- Edges: $E = 12$
- Faces: $F = 6$

Substituting into Euler's formula:

$$8 - 12 + 6 = 2$$

Therefore, the formula is verified.

6.2 Pythagorean Theorem in Three Dimensions (Space Diagonal)

Theorem

For a cuboid with side lengths a , b , and c , the space diagonal is given by:

$$d = \sqrt{a^2 + b^2 + c^2}$$

This theorem extends the classical Pythagorean theorem into three-dimensional space.

Proof

First, consider the diagonal of the rectangular base. By applying the Pythagorean theorem in two dimensions, we obtain:

$$d_1 = \sqrt{a^2 + b^2}$$

Now, the space diagonal forms a right triangle with d_1 and the height c . Applying the Pythagorean theorem again:

$$d = \sqrt{d_1^2 + c^2}$$

Substituting the value of d_1 :

$$d = \sqrt{a^2 + b^2 + c^2}$$

Thus, the formula for the space diagonal is proved.

Example

For a cuboid with:

- $a = 3$
- $b = 4$
- $c = 12$

The space diagonal is:

$$d = \sqrt{3^2 + 4^2 + 12^2}$$

$$d = \sqrt{9 + 16 + 144}$$

$$d = \sqrt{169}$$

$$d = 13$$

7. SOLVED APPLICATIONS HIGHLIGHTING SPATIAL SKILLS

Solved problems in solid geometry are an important tool for relating abstract formulas to spatial knowledge. Many problems require not only numerical calculation but also strong visual-spatial skills, such as visualizing three-dimensional objects, understanding cross-sections, and visualizing curved

surfaces. The following examples illustrate how spatial skills can be used to solve problems in university-level solid geometry.

Problem 1: Cone Volume

A right circular cone has a radius of $r = 5$ units and a height of $h = 12$ units. The volume of a cone is obtained as one-third of the volume of a cylinder with the same base and height.

Formula for the volume of a cone:

$$V = \frac{1}{3}\pi r^2 h$$

Substituting the given values:

$$V = \frac{1}{3}\pi(5^2)(12)$$

$$V = \frac{1}{3}\pi(25)(12)$$

$$V = 100\pi$$

Thus, the volume of the cone is:

$$100\pi \text{ cubic units}$$

This problem highlights the need to visualize the cone as a three-dimensional solid related to a cylinder.

Problem 2: Cross-Section Visualization

A cube is sliced by a plane passing through three vertices that do not lie on the same face. The cross-section created inside the cube is an equilateral triangle.

Such problems are very difficult in terms of visualization because the plane slices the cube diagonally, cutting three edges at the same time. Students need to visualize how a plane slices a three-dimensional object and identify the shape that is formed.

Cross-section problems are more difficult for students with poor visualization skills because the cross-section is not visible from the outside diagram of the cube.

Problem 3: Sphere Surface Area

A sphere has radius $r = 7$ units. The surface area of a sphere represents the total curved outer area enclosing the solid.

Formula for surface area of a sphere:

$$A = 4\pi r^2$$

Substituting the radius:

$$A = 4\pi(7^2)$$

$$A = 4\pi(49)$$

$$A = 196\pi$$

Therefore, the surface area of the sphere is:

$$196\pi \text{ square units}$$

This problem emphasizes the importance of understanding curved surfaces, which cannot be represented fully in two-dimensional drawings.

8. CONCLUSION

Solid geometry is one of the most challenging areas of mathematics that requires students to think logically about objects and their relationships in three-dimensional Euclidean space. In contrast to plane geometry, solid geometry requires intricate spatial relationships that require high levels of visualization, mental manipulation, and the ability to interpret two-dimensional representations of three-dimensional objects. The results mentioned in this paper emphasize the fact that university students tend to struggle with solid geometry in areas like mental rotation, interpretation of diagrams, spatial transformations, and the effective use of volume and surface area formulas. The paper shows that these issues are closely linked to visual-spatial skills, which are critical to the effective representation of internal structures of solids, understanding the hidden parts of geometric figures, and performing complex reasoning tasks like cross-section analysis. The basic theorems of solid geometry,

such as Euler's polyhedron formula and the three-dimensional Pythagorean Theorem, also emphasize the need for logical and spatial reasoning in mastering solid geometry concepts. The solved examples related to cones, spheres, and polyhedral solids also illustrate the need for mathematical computation to be complemented by conceptual understanding of spatial relationships.

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