

## MATHEMATICAL MODELING OF MAGNETOHYDRODYNAMIC NON-NEWTONIAN FLUID FLOW

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### ABSTRACT

Non-Newtonian magnetohydrodynamic (MHD) flow Non-Newtonian fluids Non-Newtonian fluids magnetohydrodynamic flow is used in very diverse applications within engineering, industrial, and biomedical contexts, including polymer processing, metallurgical operations, geothermal power plants, electromagnetic pumping, under a magnetic field blood flow, and microfluidic technologies. A non-Newtonian electrically conducting fluid with uniform transverse magnetic field is developed into a detailed mathematical model in the above work to examine the steady and incompressible flow of the fluid. The model is founded on the incorporation of non-Newtonian relation of constitutive to the equations of momentum in order to describe the nonlinear rheology of the fluid. A series of approximations are made to derive coupled governing equations of transport of energy and momentum in the case of a boundary layer and transforms them to a non-dimensional form. It is through this process that a few important controlling parameters are produced, such as the magnetic parameter, power-law index, Prandtl number, and Eckert number, or the relative importance of the electromagnetic forces, rheology of the fluid and thermal impact. The resultant nonlinear system is analyzed in regards to its physical importance as well as parametric control of the velocity and temperature fields. The model presented offers a sound theoretical basis that can be further developed into the analytical and numerical approaches to the solution of the practical magnetohydrodynamic non-Newtonian fluid flow issues.

**Keywords:** Magnetohydrodynamics, Non-Newtonian fluid, Mathematical modeling, Boundary layer flow, Magnetic field

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### Introduction

The dynamics of magnetic fields in fluids that are conductive to electricity, often simply called magnetohydrodynamics (MHD), has been an object of active scientific study since it is an incredibly complex theoretical problem with a broad range of practical applications. MHD phenomena occur when conductive fluid is moved due to the effect of an external magnetic field, and the electromagnetic forces are generated that can significantly change the movement and thermal behavior of the fluid. Simple Newtonian fluids including liquid metals, molten salts and electrolytic solutions were the focus of early research in MHD due to their comparatively simple rheological behaviour and direct relevance to fields like astrophysics, nuclear engineering and metallurgical processing. These classical works formed the basis of the study of the interaction of fluid mechanics with electromagnetism.

However, the law of viscosity of Newton is not applicable to many fluids present in industrial, technological, and biological systems. These liquids normally called non-Newtonian fluids bear nonlinear relationship between shear stress and the shear rate. Industrial slurries, paints, polymer solutions and melts, drilling fluids, foods, blood and other biofluids, are all examples. These materials

are rheologically-complicated and consequently, they are said to have shear thinning, shear thickening, viscoelastic and yield stresses. Combination of microstructure in the fluid, nonlinear viscosity, and Lorentz forces makes the flow behavior of non-Newtonian fluids in the presence of magnet fields to be very complex. Such complexities bring about the need of sophisticated theoretical and mathematical methods to properly predict and manage such flows.

Non-Newtonian fluid flow has become a growing area of research within MHD fluid flow, and has also become important in a number of emerging high-impact applications. Magnetic fields can be applied in metallurgical and materials processing industries where they are used to regulate the flow of molten metals, reduce turbulent flow, and increase the rate of heat transfer. In nuclear reactor technology MHD principles are used in liquid-metal cooling in order to enhance thermal efficiency and operational safety. The interaction of electrically conducting biological fluids with magnetic fields has been applied in biomedical engineering in magnetic drug targeting, hyperthermia therapy and diagnostic apparatus. Also, the models of MHD non-Newtonian flows can be used in polymer extrusion processes, electromagnetic pumping processes, in geothermal energy retrieval processes and microfluidics systems, in which detailed control of intricate fluids is needed.

Although of practical importance, experimental studies of non-Newtonian flows of the MHD are usually difficult, expensive, or even inconvenient especially when high temperatures or strong magnetic fields, or when the geometry involved is at a micro scale. Consequently mathematical modeling has become an effective and essential instrument towards comprehending the underlying mechanisms that drive such flows. With proper assumptions and constitutive modeling, mathematical formulations help a researcher isolate critical physical parameters, investigate parametric relationships, and predict flow and heat transfer properties with a high degree of accuracy. In addition, analytical and numerical solutions of these models make interesting insights, which are used in designing experiments and technological advancement.

### **Physical Model and Assumptions**

In order to develop a mathematically responsive and physically valuable model of magnetohydrodynamic flow of a non-Newtonian fluid, a collection of well-developed assumptions is embraced. The fluid in question is taken to be incompressible, viscous and electrically conducting which is suitable to several industrial fluids, polymeric solutions, biological fluids and liquid metals that are utilized in MHD applications. Incompressibility assumption enables neglect of changes in density thus simplifying the continuity equation and the electrical conductivity allows interaction between the fluid motion and applied magnetic field by the production of electromagnetic forces.

The flow is assumed to be two-dimensional, steady and laminar. The steady-state assumption is that all flow quantities do not depend on time, which is natural in a fully developed flow or slowly varying flows. Laminar flow conditions are assumed to circumvent the complications of turbulence modeling and are especially applicable to a range of flow regimes in the surface layer, microfluidics, and low Reynolds number flow regimes. It uses the two dimensional approximation by assuming that the dynamics of the third direction of space are insignificant to the dynamics of the primary flow and transverse directions, which is a usual method in the analysis of boundary layers and channel flows.

A uniform strength of magnet field is applied in the direction which is perpendicular to the flow of fluids. It is presumed that the field of magnetism is in right angles to the direction of the flow a Lorentz force is generated acting in a direction opposite to the movement of the fluid hence changes the velocity distribution. The magnetic Reynolds number is assumed to be relatively small to ensure that the induced

magnetic field due to the flow of the fluid does not become significant as compared to the applied magnetic field. This is true in the case of moderate electric conductivity flows and comparatively low characteristic velocities, and greatly simplifies the electromagnetic coupling since it does not require solving Maxwell equations to get the induced field.

The current model assumes the body forces, i.e., the gravitational and buoyancy forces, are not taken into account and the only body force is the electromagnetic body force (Lorentz force) that occurs due to the interaction between an electric current and the magnetic field. This is the right assumption when an electromagnetic effect dominates the flow behaviour or when the flow is horizontal and the effect that buoyancy by thermal means is negligible. Moreover, the effects of thermal radiations, chemical reactions, and phase change are not considered, unless explicitly mentioned, which means that the analysis is then focused on the interaction of non-Newtonian rheology, magnetic field, and heat transfer as the main factors.

A power-law non-Newtonian constitutive relation is used to describe the rheological behaviour of the fluid, and thus, nonlinear viscosity effects, including shear thinning and shear thickening, are captured. The model is very popular because it is mathematically easy and also because it can model a wide range of non-Newtonian fluids that are found in practice. The current formulation takes into consideration the role of fluid microstructure in the momentum transport whilst retaining its analytical and numerical tractability through the inclusion of the power-law model.

### Governing Equations

Mathematically, magnetohydrodynamic flow of the non-Newtonian fluid is developed by using the fundamental concepts of the principles of conservation of mass, momentum and energy. These equations of state provide a complete description of the flow and the thermal action of an electrically conducting and non-Newtonian fluid that is a flowing fluid subject to an external magnetic field. This model is developed based on assumptions concerning a steady, laminar and incompressible, two-dimensional flow are often assumed in the analysis of the boundary layer and which is appropriate in a very large variety of engineering and biomedical problems where the velocity of flow is intermediate, and the length scales involved are intermediate.

The gel is assumed to be electrically conducting and placed in a uniform transverse magnetic field and perpendicular to the primary flow. Coupling of the magnetic field with the fluid motion creates an electromagnetic body force known as the Lorentz force and has the opposite direction of both the fluid motion and the field of action can be considered to be of importance in the transportation of momentum in the flow field. This is an electromagnetic force which has been explicitly included in the momentum equation and plays a role in the regulation of both distribution of velocity and the formation of the boundary layer.

Appropriate constitutive relation is used to model the non-Newtonian behavior of the fluid with account of the nonlinear dependence of the shear stress on the velocity gradient. As opposed to Newtonian fluids, in which viscosity is constant, the effective viscosity of a non-Newtonian fluid is not constant but changes depending on deformation rate, which gives rise to complicated flow behaviour. To include this constitutive relation in the governing equations adds further nonlinearity that is necessary to represent the realistic fluid behavior when acting under the influence of a magnetic field.

The flow arrangement is described using a Cartesian coordinate system as the xxx-axis is oriented along the major direction of the fluid flow and the yyy-axis is perpendicular to the flow. The corresponding

velocities on the  $x$ - and the  $y$ -directions are represented by  $u$  and  $v$ , respectively. The coordinate system is very useful in the study of the boundary layer, as the equations governing a boundary layer can be simplified without losing the key physical characteristics of the problem using this coordinate system. These equations and coordinate frames lead to the formulation of a coupled system of nonlinear partial differential equations that govern the conservation of mass, momentum and energy of magnetohydrodynamic non-Newtonian fluid flow.

### Continuity Equation

In the case of the incompressible fluid, the principle of mass conservation holds that the mass flowing into any small fluid mass as a whole must be zero. This state gives rise to continuity equation which represents mathematically the balance between the components of velocity in the flow field. The continuity equation in the current two-dimensional flow setup is given in terms of within the current local flow setup.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

In which  $u$  and  $v$  are the components of velocity perpendicular to the  $x$ - and  $y$ -direction, respectively? This equation means that a spatial change in the streamwise velocity should be followed by an equal change in the transverse velocity to ensure the fluid density remains the same. The continuity equation is thus a major restriction on the velocity field and also, the conservation of mass within the flow field. Boundary layer modeling also uses it as the basis of applying stream functions or similarity transformations which simplify the governing equations without compromising the physical constraint of incompressibility.

### Momentum Equation

The  $x$ -direction momentum equation is the equation that keeps the forces that are exerted on the non-Newtonian fluid under the influence of an applied magnetic field. It is expressed as:

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial y} (\tau_{xy}) - \sigma B_0^2 u,$$

where  $\rho$  is the density of a fluid,  $\sigma$  is the electrical conductivity of a fluid and  $B_0$  is the strength of an external magnetic field. The inertial forces on the left-hand side of the equation are the streamwise and transverse acceleration of the fluid. The unit in the right-hand-side is diffusion of momentum due to viscous and non-Newtonian force that is expressed as shear stress component  $\tau_{xy}$ . The final one is an examination of the electromagnetic force of body which also goes by the name Lorentz force that develops between the induced electric current and the applied magnetic field. This is the opposite of the flow movement of force and is most likely to slow down the velocity, thus increasing the resistance to the flow. The resulting effects of inertia, non-Newtonian rheology and magnetic effects on the momentum transport of non-Newtonian fluid flow in the presence of a magnetic field can be explained with the combination of these terms (magnetohydrodynamic flow).

### Non-Newtonian Constitutive Relation

In order to determine the rheological behavior of the non-Newtonian fluid, a power-law constitutive model is used in order to establish a non-linear relationship between the shear stress and the velocity gradient. As per this model, the shear stress component:

$$\tau_{xy} = K \left( \frac{\partial u}{\partial y} \right)^n,$$

K is the consistency index that determines the apparent viscosity of the fluid and  $n$  is the index of flow behaviour that determines the nature of flow behaviour i.e. non-Newtonian behaviour. The fluid is considered to be shear-thinning in the instance where the  $n < 1$ , i.e. the effective viscosity declines as the shear rate increases and this effect is normally observed in polymer solutions and also in the biological fluids. With  $n=1$ , the model reduces to the classical Newtonian fluid case, where the viscosity is constant, but with  $n > 1$  are the shear thickening cases, where the apparent viscosity increases with shear rate, as in some suspensions and dense slurries. Substituting this constitutive relation by the momentum equation introduces high nonlinearity in the governing equations that represents the concerted action of both the variable viscosity and the magnetic fields on the flow field. This is a nonlinear formulation that is necessary to describe the realistic behaviour of magnetohydrodynamic non-Newtonian fluid flow and the foundation of later analytical or numerical analysis.

### Energy Equation

The energy equation controls the thermal behavior of the non-Newtonian fluid under the influence of a magnetic field and it is the conservation of energy in the flow field. Considering viscous dissipation effects, the energy equation becomes as below:

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \tau_{xy} \frac{\partial u}{\partial y},$$

$T$ , means the temperature of the fluid,  $c_p$  is the specific heat constant pressure and  $k$  is the thermal conductivity of the liquid. The left side of the equation has been identified with the convective transfer of the thermal energy along the direction of streams and transversal direction alongside the fluid motion. The former in the right-hand side is the heat conduction of the fluid and the latter is the dissipation of a fluid due to viscosity which is the conversion of mechanical energy to thermal energy by the internal friction caused by the deformation of the fluid. In non-Newtonian fluids, viscous dissipation may be particularly significant because of nonlinear correlation between shear rate and shear stress. The introduction of this term enables the model to seek the effects of rheology and the intensity of flow on the temperature distribution so that the energy equation is imperative in the analysis of the heat transfer properties within the magnetohydrodynamic non-Newtonian flows.

### Boundary Conditions

In order to get physically relevant and mathematically well-posed response of the governing equations, suitable boundary conditions are applied both on the solid surface and in the free stream region distant to the wall. The conditions are used to indicate the physical limitations of the flow system and make sure that the velocity and temperature fields are uniquely defined in the computational domain.

On the solid boundary, which is given by  $y=0$ , the no-penetration and no-slip conditions are applied classically. As per the no-slip condition, the velocity of the fluid across the stream through direction is

assumed to be zero,  $u=0$ , which means that fluid in immediate contact with the wall sticks to the stationary surface. At the same time the component of the transverse velocity is considered as zero,  $v=0$ , which assumes that mass flux across the solid surface is zero. These are physically realistic conditions of viscous fluids and are common in the study of the boundary layer and channel flows.

Besides the velocity limits, there exists a thermal boundary condition that is given at the wall. The temperature of the wall is set to that of a constant,  $T=T_w$ , which is an isothermal surface. This condition creates a temperature difference between the wall and the fluid surrounding the wall, which causes heat transfer across the boundary layer. Constant wall temperature is a rather common assumption in theoretical and applied studies of heat transfer, because it is easy to treat analytically and numerically, and it is also a fairly accurate approximation of many real-world engineering problems.

At a great distance of the wall, that is, when  $y$  tends to infinity, the effect of the solid boundary decreases, and flow is said to assume the state of free stream. The velocity of the fluid in this area will tend to the typical free stream velocity as an asymptote and the temperature of the fluid will be similar to that of the surrounding fluid. These asymptotic edge conditions make sure that there is a smooth transition of the boundary layer to the external flow and fully specify the mathematical model of the problem.

### **Mathematical Characteristics of the Model**

The mathematical model that has been developed to model magnetohydrodynamic non-Newtonian fluid flow has some salient features which highly affect both the analytical structure and numerical behavior of the model. These properties are mainly due to the complexity in the form of non-Newtonian fluids, the existence of electromagnetic forces created by the applied magnetic field and the interaction of mechanisms of transport of momentum and energy. Nonlinear constitutive relations and coupled field equations characterize the current formulation, as opposed to classical Newtonian fluid models, making the problem much more difficult to solve mathematically.

The sources of the nonlinearity of the model are various, such as the convective acceleration terms in the equations of momentum and energy and nonlinear dependence of shear stress on the velocity gradient that is brought by the non-Newtonian constitutive relation. This nonlinearity is a reflection of the real-life behavior of most practical fluids but also denies the existence of closed-form analytical solutions. Consequently, it can be seen that specialized methods of analysis like perturbation methods, homotopy based methods or even numerical methods are commonly needed to get reliable solutions. The nonlinearity of the equations also means that minor fluctuations in the controlling parameters can result in great changes of the flow and thermal fields, and physical conditions are sensitive to the flow.

The close interrelation between the momentum and energy equations is also another significant characteristic of the model. The convective effect of heat transportation is directly determined by the velocity field, whereas the shear-sensitive viscous dissipation phenomenon in the energy equation is determined by velocity gradients obtained after the momentum balance. Moreover, the induction magnetic field has its effect on the temperature distribution indirectly through the alteration of the velocity field and internal generation of heat by the electromagnetic damping effects. This interdependence requires a joint and steady resolution of the underlying equations so as to fully resolve the coupled nature of fluid movement and heat exchange.

### **Nonlinearity of the Governing Equations**

The equations that govern the current magnetohydrodynamic non-Newtonian fluid model are nonlinear in nature because of a number of reasons. The first is the quadratic nonlinearities created by the convective acceleration terms which occur in both the energy and momentum equations due to the product of velocity components and their spatial derivatives. These parameters reflect the movements of momentum and heat by the fluid motion and they are a major source of nonlinearity even in classical Newtonian flow problems.

Second, there is the nonlinear dependence on the velocity gradient of shear stress due to the adoption of a power-law constitutive relation to represent the non-Newtonian behavior of the fluid. In contrast to Newtonian fluids, shear stress in a power-law model changes not at a linear rate with the strain rate, but is a nonlinear response that grows increasingly large as the index of flow behavior becomes nonunitary. This is especially at shear-thinning ( $n < 1$ ) and shearthickening ( $n > 1$ ) regimes, in which the apparent viscosity of the fluid can change significantly due to changes in shear rate.

Moreover, the viscous dissipation term added to the equation of energy also increases the nonlinearity of the system. The term directly relates the temperature field to the gradients of velocity, or the transformation of mechanical energy into thermal energy as a result of friction within the body. Consequently, the temperature field becomes highly coupled with the nature of the flow, and this introduces an extra level of non-linearity between the equations of momentum and energy. Due to these nonlinear combined effects, the equations of governing variables typically do not have closed-form analytical solutions. As a result, their analysis normally involves application of a strong analytical tool or strong numerical means to find the right and physically significant solutions.

### **Coupling between Momentum and Energy Equations**

Another mathematical feature of the current model is the high level of interaction between the equations of momentum and energy, which is very important in defining the overall flow and thermal properties. The velocity field which is determined using the momentum equation has direct effect on the temperature distribution because of the convective terms of heat transport in the energy equation. Fluctuations in the streamwise and transverse components of velocity influence the velocity of thermal energy transport in the fluid, thus, defining the thermal boundary layer and thermal gradients.

The energy equation on the other hand is also coupled backwards to the momentum field via the viscous dissipation term which is the shear stress and velocity gradients, governed by the momentum equation. This coupling is further enhanced in fluids which are non-Newtonian, as the effective viscosity is proportional to the shear rate, thus the rheological behaviour of the fluid is nonlinearly coupled to both the momentum and thermal transport processes. Consequently, variations in the velocity field may produce a tremendous influence on the temperature distribution, and the reverse may also happen.

Moreover, the magnetic field exerted follows the indirect effect on the temperature field by changing the distribution of velocity via the Lorentz force and by participating in internal heating in the form of Joule heating. This electromagnetic effect increases the interdependence of the momentum equation and the energy equation, and therefore there is the need to consider them as a coupled system. Therefore, the combination of the equations of momentum and energy is needed to provide the correct representation of the flow and heat transfer properties because decoupled or sequential methods can be unable to represent the complex interactions of magnetohydrodynamic non-Newtonian fluid flow.

### Physical Consistency and Model Validity

The mathematical model suggested is physically sound in the context of the principles of fluid mechanics and electromagnetism given the mentioned set of assumptions. Continuity equation, and momentum equation and energy equation are the equations that guarantee preservation of mass, and take proper consideration of inertial, viscous, electromagnetic, and thermal influences. The induced magnetic field is not taken into account because the assumption of low magnetic Reynolds number is valid in cases of flows where electrical conductivity and velocity scales are modest. Likewise, boundary layer approximation is applicable to flows that are described by high aspect ratios and components of streamwise velocities that are dominant. Despite the simplified character of the power-law model of non-Newtonian behavior, the model represents the rheological behavioral characteristics of several practical fluids. The model is therefore a credible and mathematically sound model to study magnetohydrodynamic non-Newtonian flow of fluids and can form a base to further extensions with other physical effects or more complicated constitutive relationships.

### Discussion of Physical Parameters

The complex interplay of electromagnetic forces, fluid rheology and thermal transport mechanism controls the behavior of magnetohydrodynamic non-Newtonian fluid flow. The magnetic parameter is one of the controlling parameters that has dominant role in that it creates a Lorentz force which acts contrary to the direction of fluid motion. This is a resistive force, which occurs as a result of the interaction between the induced electric current and the magnetic field applied and essentially averts the fluid velocity. Momentum boundary layer grows with increase in the magnetic parameter, and the total flow is slowed down. It is a strong magnetic damping process especially in fluid with electrical conductivity and finds extensive use in industrial processes to regulate flow stability and minimise turbulence. Also, a powerful magnetic field increases thermal energy in the fluid because of the Joule heating which is caused by electrical resistance and leads to an increase in temperature across the entire thickness of the boundary layer.

The non-Newtonian parameter, which is denoted by the power-law index  $n$ , affects the distribution of velocities and shear stress behavior strongly, in particular in the near-wall region. In the case of shear-thinning fluids ( $n < 1$ ), The apparent viscosity reduces as the shear rate increases, which causes lower flow resistance and increased velocities than in Newtonian fluids. The effect of this behavior is the development of a thinner velocity boundary layer and increased momentum transport. On the contrary, shear-thickening fluids ( $n > 1$ ) have a higher apparent viscosity with shear rate that increases shear resistance to deformation and dramatically slows the velocity of the fluid. This results in an increase of the momentum boundary layer, and the velocity gradients along the wall weaken. These differences demonstrate the importance of non-Newtonian rheology in the flow structure and also the need to use relevant constitutive models in the MHD flow.

Prandtl number is a ratio of the momentum diffusivity to the thermal diffusivity which is a very powerful factor on the thermal transport characteristics. Oils and polymer melts with large values of Prandtl numbers possess much lower values of thermal diffusivity resulting in thinner thermal boundary layers and steeper temperature gradients on the wall. Conversely, fluids with low Prandtl numbers, e.g. liquid metals, possess higher thermal diffusivity and so the thermal boundary layer is thicker and temperature distributions are more uniform. The Eckert number is another parameter that defines the thermal behavior, which measures the conversion of the kinetic energy to the internal energy in the form of the viscous dissipation. The larger Eckert numbers give greater effect to viscous heating which is especially significant in high-speed flows, high-viscosity fluids, and systems of high shear. Viscous dissipation in

these instances makes a significant contribution to the heating in the fluid and can no longer be ignored in thermal analysis.

In general, the interaction of the magnetic parameter, index of power-law, Prandtl number, and Eckert number is what we refer as coupled velocity and temperature fields of the magnetohydrodynamic non-Newtonian fluid flow. The magnitude of these parameters is significant to understand how to maximize the functioning of flow control, heat transfer, and energy cost in the real engineering system environment.

## Conclusion

This paper has presented a detailed mathematical model of the magnetohydrodynamic flow of a non-Newtonian fluid when a transverse magnetic field is applied on it. This formulation is founded on the underlying conservation laws of mass, momentum and energy where appropriate assumptions are made which are realistic physical conditions. The presence of a power-law constitutive relation is what allows the model to describe key non-Newtonian rheological properties, including shear thinning and shear thickening, whereas the presence of electromagnetic effects explains the effect of the Lorentz force on fluid flow and thermal behaviour.

It was also determined that through non-dimensionalization, some important dimensionless parameters that were used to determine the flow and heat transfer properties were discovered, such as the magnetic parameter, power-law index, Prandtl number, and Eckert number. A qualitative examination of these parameter determines that they have a great influence on the velocity suppression, development of a boundary layer, and the temperature distribution in the flow domain. The findings reveal that magnetic fields can successfully be implemented as a control measure of regulating the movement of fluids and thermal performance, especially in non-Newtonian fluids that are electrically conducting.

The mathematical system devised offers a solid theoretical base to conduct additional analytical and numerical studies. The model can easily be generalized to unsteady flow conditions, alternative non-Newtonian fluid models that include Casson, Carreau, or viscoelastic fluids, as well as the addition of other physical effects, such as thermal radiation, chemical reactions, mass transfer and porous media. Therefore, the current work can be added to the overall knowledge of magnetohydrodynamic non-Newtonian fluid physics and provide useful information on designing and optimization of high-technology engineering and biomedical systems of complex fluids under a magnetic field.

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