IJAER/November-December 2017/Volume-6/Issue-3 ISSN: 2278-9677

International Journal of Arts & Education Research

STUDY ON THE EVOLUTION OF LINEAR ALGEBRA AND ITS EFFECT ON SYSTEM OF EQUATIONS

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ABSTRACT

In numerous frameworks hypotheses, the recognizable proof and control boundaries, the goal of multivariate polynomial frameworks, and polynomial advancement issues show up as focal issues. Generally, techniques for explaining polynomial conditions have been created in the field of mathematical calculation. It is known as one of the most out of reach zones of arithmetic, in spite of the huge measure of writing accessible. In this article, we present a technique for unraveling frameworks of polynomial conditions utilizing just mathematical direct polynomial math and frameworks hypothesis apparatuses, for example, acknowledgment hypothesis, SVD/QR, and eigenvalue figuring's. By isolating the coefficients and monomials in a grid of coefficients duplicated by a base of monomials, the main job moves into the field of direct polynomial math. The use of the hypothesis of acknowledgment to the structure dependent on monomials permits to discover all the arrangements of the framework from the computation of the eigenvalues. The arrangement of a polynomial advancement issue compares to an outside eigenvalue issue. During ID and control, significant applications are discovered, for example, worldwide streamlining of organized least squares issues.

Keywords: *Evolution, Effects, Mathematical*

INTRODUCTION

The polynomial system solution is typically studied in the field of algebraic geometry where it was the primary problem of attention until the end of the 19th century. Algebraic geometry was transformed into abstract algebra in the 20th century, and polynomial system resolution came into focus again around the 1960s with the seminal work of B. Buchberger on the basis of Gr obner algorithms giving rise to the field of computer algebra. The Gr obner basic solution approach is still dominant in polynomial system resolution, but suffers from poor numerical properties because it relies on symbolic operations and accurate arithmetic. The solution of polynomial equation systems is still a relevant task, which can be seen in a multitude of scientific and engineering applications.

In practical situations, floating-point arithmetic is desired and stable numeric values are of paramount importance. For these reasons, Grower's basic approaches are not appropriate. The current article addresses these issues by looking at polynomial system resolution as a linear algebra task. Linear algebra appears to be a natural setting for the solution of the polynomial system: the root of the problem has strong ties to the linear algebra, see Sylvester and Macaulay 35, 36]. We will develop a linear algebra-based root-finding method starting from the work of these early ‘linear' algebraists.

The method is developed in the modern language of numerical linear algebra and dynamic system theory. From the numerical linear algebra, we borrow the notions of row / column spaces and null spaces, as well as the tools used to solve homogeneous linear systems, peculiar value problems and singular value decompositions. From dynamic systems theory, we use the realization theory tools: we will study the null space of the Macaulay coefficient matrix, which has an observability matrix structure. Applying the theory of realisation will lead to the fact that the root-finding problem can be solved as a problem of own-value. It should be noted that many of the elements presented are not new; similar ideas have been described in among others. Yet, surprisingly, the linear algebra approach to polynomial system resolution has remained largely unknown, perhaps due to the fact that key results are scattered across literature and, to the best of authors' knowledge, have never been compiled into a conceptually simple framework.

We begin with a brief overview of the relevant concepts that we borrow from numerical linear algebra and dynamic system theory.

OBJECTIVE OF THE STUDY

1. The study of polynomial equations addressing and polynomial enhancement problems emerges as focal assignments in framework identification and control settings.

2. Coefficient matrix, which ends up containing all the information that is important for the arrangement of equations.

Numerical Linear Algebra

The second half of the 20th century witnessed the maturation of the numerical linear algebra, which has become a well-established field of research. A multitude of reliable linear algebra numerical tools is well understood and developed. We will outline an independent value-based solution method in which linear algebra notions such as row and column space, zero space, linear (in) dependence and matrix rank play an important role, as well as essential linear algebra-based numerical tools such as singular value decomposition and self-value decomposition. The central object of the proposed method is the Macaulay matrix a Sylvester-like structured matrix based on the coefficients of the multivariate polynomial set.

The Macaulay matrix is obtained by considering multiplications (shifts) of the monomial equations so that the result has a certain maximum degree. Translates a system of polynomial equations into a system of homogeneous linear equations: the polynomials are represented as (rows of) the Macaulay matrix multiplied by a multivariate Vander monde vector containing the monomials. The proposed method proceeds by iterating by increasing the degree to which the Macaulay matrix is built. We will study the dimensions, rank and (co-)rank1 of the Macaulay matrix to an increasing degree. The Macaulay matrix will be over-determined to a certain degree. Interpreting this as a homogeneous system allows us to divide the unknowns (monomials) into linearly independent and linearly dependent unknowns. We will see that the corank of the Macaulay matrix corresponds to the number of linearly independent monomials and, furthermore, is equal to the number of solutions.

System Theory

We will borrow elements from system theory in general and from the theory of realization in particular. Realization theory will enter the scene when we analyse the null space structure of the Macaulay matrix. The link between the theory of realisation and the polynomial systems should not come as a surprise. For one-dimensional LTI systems, it is well known that the Sylvester matrix of the transfer function polynomial denominator is the left annihalator of the LTI system observation matrix. The theory of realisation of multidimensional (nD) systems has been linked to polynomial system resolution in using the Gr obner base view. In multidimensional dynamic systems, dynamics do not depend on a single independent variable (such as discrete time in linear time-invariant systems modelled by difference equations), but depend on several independent variables (such as space and time in PDEs). We will show that the null space of the Macaulay matrix can also be modelled as the output of an autonomous multidimensional (nD) system, possibly a singular (i.e. a descriptor) system. The null space of the Macaulay matrix has a multiplication structure that stems from the monomial structure of the unknown. By combining the structure of multiplication in the null space with the interpretation of the theory of realization, we develop an algorithm to create a (generalized) problem of self-value that delivers all the roots of the system.

Solving polynomial systems has been central throughout the history of mathematics and has been almost synonymous with algebra until the beginning of the 20th century. Sylvester and Macaulay were the first to use root-finding phrases in the (premature) linear algebra framework. This point of view is at the heart of the proposed matrix approach. Unfortunately, these contributions were overshadowed by a shift to abstract algebra in the early 20th century, and were abandoned for almost a century.

Around the 1960s, the development of the Buchberger Gr obner Basis Algorithm again focused on computational algebra, together with the advent of digital computers. Although Gr obner bases have dominated computer algebra ever since, the inherent exact arithmetic (i.e. symbolic calculations) of Buchberger's algorithm has made its extension to floating point arithmetic cumbersome; a limited number of alternative approaches are available not surprisingly, often involving (numeric) linear algebra.

Khovanskii's theorem on fewnomials

Polynomial equations emerge in numerous mathematical models in science and building. In such applications one is ordinarily keen on arrangements over the real numbers $\mathbb{R}$ i Rather than complex numbers. This study of the real roots of polynomial systems is significantly more problematic than the study of complex roots. Indeed, even the most fundamental investigations remain unanswered to date. Let us begin with a sound investigation of this kind: Question 3.6. What is the most extreme number of confined real roots of any arrangement of two polynomial equations in two variables each of which has four terms? The polynomial equations considered here are similar

$$\begin{aligned} f(x,y) &= a_1x^{u_1}y^{v_1} + a_2x^{u_2}y^{v_2} + a_3x^{u_3}y^{v_3} + a_4x^{u_4}y^{v_4}, \\ g(x,y) &= b_1x^{\tilde{u}_1}y^{\tilde{v}_1} + b_2x^{\tilde{u}_2}y^{\tilde{v}_2} + b_3x^{\tilde{u}_3}y^{\tilde{v}_3} + b_4x^{\tilde{u}_4}y^{\tilde{v}_4}. \end{aligned}$$

Where a_i, b_j are arbitrary real numbers and u_i, v_j, u_i, v_j are arbitrary whole numbers. To remain reliable with our prior conversation, we will check just arrangements (x, y) in $(\mathbb{R}^*)^2$ That is, we need both x and y to be non-zero real numbers. There is a conspicuous lower bound for Question 3.6: thirty-six. It is anything but difficult to record the arrangement of the above form, which has 36 real roots:

$$f(x) = (x^2-1)(x^2-2)(x^2-3) \quad \text{and} \quad g(y) = (y^2-1)(y^2-2)(y^2-3).$$

Every one of the polynomials f and g relies upon one variable in particular, and it has 6 non-zero real roots in that factor. In this way the framework $f(x) = g(y) = 0$ has 36 distinct isolated roots in $(\mathbb{R}^*)^2$. Note, in addition, that the developments of f and g have, as required, four terms each. It is not clear from the earlier that Question 3.6 even bodes well: for what reason should such an extreme exist? There's absolutely no off chance that we're thinking about complex zeros, as one can self-assertively get numerous complex zeros by expanding the degrees of the equations. The fact is that such an unbounded increase in roots is unthinkable over real numbers. This has been demonstrated by Khovanskii. He found a bound on the quantity of real roots that does not depend on the degrees of the equations in question. We're expressing the version for positive roots.

Babylonian sources contain many examples of quadratic equations of ancient times. Many old Babylonian tablets have extensive lists of quadratic problems. The normal form is x + y = b and xy = a. These Babylonian problems suggest that the Babylonians originally wanted to deal with the relationship between the area and the perimeter of the rectangle. In ancient times, many believed that the area of the field depended solely on its perimeter. It is conceivable that the Babylonian scribes, in order to show the rectangles of the same perimeter, could have very different areas, constructed tables of area'a' for the given perimeter 2b using different values of length x and width y. Study of such tables concerning the varying lengths in the given table

in [36]: $x = \frac{b}{2} + z$ and widths $y = \frac{b}{2} - z$ to the areas $a = \left(\frac{b}{2} + z\right)\left(\frac{b}{2} - z\right) = \left(\frac{b}{2}\right)^2 - z^2$

Then $z = \sqrt{\left(\frac{b}{2}\right)^2 - a}$

Therefore, $x = \frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - a}$, $y = \frac{b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - a}$

The Babylonians have not given a formula. They presented each problem with the number assigned to the length, width and area. Specific numerical calculations which could be interpreted in the form referred to above are indicated.

The contribution of the Babylonians was no less important than that of the Egyptians. But, like the Egyptians, the Babylonians were not a science. For Babylonians, the records are not so abundant that the Egyptians used papyrus. The Babylonians used a stylus with markings on clay tablets. These tablets were then baked, usually in the sun. There is nothing comparable to the papyrus among the Babylonians. The Babylonians had long maintained close commercial contact with the Egyptians. They were familiar with the decimal numbering system.

It is often said that the Babylonians were the first to solve quadratic equations. But this is a matter of simplification. What they developed was an algorithmic approach to problem-solving, which would give Ri ~ e a quadratic equation. In essence, the method is one of completing the square. All the Babylonian problems had answers, which were positive because the usual answer was a long one.

Sixteenth century mathematics

The European algebraists of the sixteenth century had progressed as far as possible in the continuation of the middle Ages Islamic algebra. They were experts in algebraic manipulation. They knew how to solve any algebraic equation of up to four degrees. Solutions have been provided in the form of rules of procedure. Most of these authors used symbolization of the unknown and its powers. But for the coefficients, there were no symbols. There is no formula in any of these algebra texts, such as the quadratic formula found in any of the current algebra texts.

During the sixteenth century, Italy was very interested in retrieving all the existing Greek mathematical works.

The basic works of Euclid, Archimedes and Ptolemy were translated several centuries earlier. However, since the translators were not experts in mathematics, their work was not always fully understood. An effort has been made to translate these works as well as other Greek mathematical works from the original Greek. These new translations have been made by mathematicians. The most important figure was the Italian mathematician Federigo Commandino (1509-1575). He prepared Latin translations of virtually all known works by Archimedes, Appolonius, Pappus, Aristarcus, Autolycus, Heron and others. With each translation, Commandino included extensive mathematical comments, clarifying difficulties and providing references from one work to other related ones.

The seventeenth century

The seventeenth century is shining well in the history of mathematics. Early in the seventeenth century, Napier unveiled his invention of logarithms. Harriot and Oughtred have contributed to the notation and codification of algebra. Galileo founded the science of dynamics, and Kepler passed his laws on planetary motion. Later in the century, Desargues and Pascal opened up a new field of pure geometry. Descartes introduced analytic geometry of the modem. Fermat laid the foundation for the theory of modem numbers. Huygens contributed to the probability theory. Towards the end of the century, Newton and Leibniz made a calculation. During the seventeenth century, many new and vast fields were opened for mathematical investigation.

Thomas Harriot (1560-1621)

Harriot's outstanding publication was published in the seventeenth century. Harriot is considered to be the founder of the English School of Algebra. His great work in the field, the Artist Analytical Praxis, was published in 1631, 10 years after his death. This is about the theory of equations. It includes the treatment of first, second, third and fourth degree equations. It includes the formation of equations having given roots, the relationship between the roots and the coefficients of the equation, the transformation of the equation into another having roots having a specific relationship with the roots of the original equation and the numerical solution of the equations.

Approximate solution of equations

Newton has introduced a method for approximating the values of the real roots of the numerical equation. This applies equally to either an algebraic or a transcendental equation. This method is modified and is known as the Newton method, which is given as follows in:

"If f(x) = 0 has only one root in the interval [a, b] and if neither j'(x) nor f'(X) disappears in this interval, and if Xo is chosen as one of the two numbers a and b for which f(xo) and f(xo) are chosen. F"(XO) has the same sign, and then $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ is nearer to the root than is x_o."

To construct a series for functions defamed implicitly, such as $y^3 + a^2 y - 2a^3 + axy - x^3 = 0$.

Newton has used the method of successive approximations. He invented it. It's called the reversion method. This equation was called a 'affected equation'

According to solve the equation y3-2y-5 = 0, he first noted that 2 was an approximate solution. Let the actual solution be represented by y = 2 + p, where p is a small amount. Substituting this in the original equation and simplifying it, he got p3 +6p2 +1Ox-1 = 0

Because p is small, he neglected the higher terms of p, and got 10p-1 = O

Then p = 0.1 is the approximate solution to the second equation. The actual solution should be p = 0.1 + q where q is another small number.

Replace p = 0.1 + q where q is another small number. Substituting 0.1 + q for pin of the second equation, the third equation was q3 +6.3q2 +11.23q +0.061 = o.

Since q is small, q3 and q2 have been neglected.

Then he was given 11.23q +0.061 = 0

Then q = -0.0054 as a solution;

Again, since q = -0.0054 is an approximate solution, the actual solution for this equation should be-0.0054 + r, where r is small.

Then the term q3 was ignored in the third equation, so the fourth equation is obtained by substituting-0, 0054 + r in the equation 6.3q2 +11.23q + 0.061 = 0.

Algebra in the 19th century

In the 1800's, Algebra meant solving equations. At the beginning of the 19th century, Carl Friedrich Gauss published the Disquisitiones Arithmeticae. In it, he discussed the basics of numerical theory. In addition to proving the law of quadratic reciprocity, he introduced several new concepts that provided early examples of groups and matrices.

Gauss' study of solutions of cyclotomic equations in the Disquisitions and Augustin-Louis Cauchy's study of permutations in 1815 helped solve the problem of algebraic equations higher than four degrees. In 1827, Niels Henrik Abel proved the impossibility of solving a general equation of degree five or higher in terms of radicals. Then Evariste Galois formed a relationship between algebraic equations and root permutation groups. The work of Galois was not published until 1846. In 1854, Arthur Cayley gave the first definition of an abstract group.

The study of ‘numbers' determined by algebraic equation solutions led to the definition of a number field. It was given by Leopold Kronecker and Richard Dedekind. Soon after Heinrich Weber gave the abstract definition of English mathematicians George Peacock and Augustus De Morgan tried to axiomatize the basic ideas of algebra. They also tried to determine exactly how many the properties of the integers could be generalised to other types of quantity. This study led to the discovery in 1843 of a quaternion by William Rowan Hamilton.

Algebra in twentieth century

The 20th century was a golden age for mathematics. It has been estimated that more mathematical research has been done in the twentieth century than in the entire preceding history of mathematics. The subject has undergone tremendous growth. The crowning achievements of the twentieth century were the solution to three celebrated challenges: Fermat's Last Theorem, the Four-Color Problem and the Kepler Conjecture. In the twentieth century, mathematics gave rise to two strands: pure mathematics and applied mathematics. The most obvious trends in pure mathematics are those of unity and abstraction. There is an inter-relationship between the various branches of modern pure mathematics. It was quite obvious in the nineteenth century whether one worked in algebra, geometry or analysis. But in the twentieth century, mathematics was not easily divided into compartments.

Topology is the most unifying branch of modern mathematics. Although it began in the nineteenth century, it developed in the twentieth century. Now it has a major influence on all branches of pure mathematics. The other main branches of pure mathematics are Abstract Algebra, Functional Analysis and Graph Theory. The theory of infmite sets plays a significant role in both topology and functional analysis.

Algebra has been associated with most of the branches of modem mathematics. The growth of the algebraic structure was a powerful influence in the twentieth century. Group Theory remains the most fundamental of the various structures considered by mathematicians. It is considered to be the supreme art of mathematical abstraction. Evariste Galois (1811-1832) was one of the great pioneers of this abstract theory. Galois has launched Group Theory for Equations. On the last night of his life, he wrote a summary of his findings in the theory of equations. It's now known as the Galois Theory. Ancient problems such as the duplication of the cube, the trisection of the angle, the solution of the cubic and biquadrate equations and the solution of the algebraic equation of any degree can be found in Galois Theory. His work was first published in 1846 by Liouville in his Journal de Mathematiques.

CONCLUSION

This summarizes the study entitled an attempt has been made in this study to shed light on the growth of algebra, particularly linear and quadratic equations. Ancient polynomial equations, mediaeval polynomial equations, early modern and modern polynomial equations and algebraic concepts of Egyptians, Babylonians, Indians, Greeks, Chinese, Arabs, Europeans, etc. examined and presented significant aspects in order to portray the historical basis of algebra spanning over a period of more than three thousand years (1650 BC-Modem period [from 1701 AD to da da]. The evolution of ideas and a number of concepts pertaining to algebra can be discerned from the accounts collected, compiled and interpreted from secondary sources. The contributions of Indian mathematicians for the period 301

AD-800 are presented below. The Bakhsh8Ji Manuscript is intended to bridge the long gap between the SulbasDtras of the Vedic period (800-200 BC) and the mathematics of the Classical period (400 AD-1200) of Indian mathematics. There are no special symbols for the unknown in the Bakhshali Manuscript. Its place in the equation remains vacant. The sign of emptiness is put there to indicate the unknown. The unknown quantity was called yadrccha or vanaha or Kamika, which means any desired quantity.

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