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STUDY ON THE CONCEPTS OF LINEAR ALGEBRA EQUATIONS

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ABSTRACT

In mathematics, the theory of linear systems is the basis and a fundamental part of linear algebra, a subject which is used in most parts of modern mathematics. Computational algorithms for finding the solutions are an important part of numerical linear algebra, and play a prominent role in engineering, physics, chemistry, computer science, and economics. A system of non-linear equations can often be approximated by a linear system (see linearization), a helpful technique when making a mathematical model or computer simulation of a relatively complex system. Assyrians, egyptians, babylonians and phoenicians are the most important people of the antiquity who have made considerable contributions to the subject. Of these, only the egyptians and babylonians have made a significant contribution to the advancement of mathematics. Information in science, business, and mathematics is often organized into rows and columns to form rectangular arrays called “matrices” (plural of “matrix”). Matrices often appear as tables of numerical data that arise from physical observations, but they occur in various mathematical contexts as well.

Keywords: *Concepts, Linear Algebra*

INTRODUCTION

Much of our knowledge of ancient Egyptian mathematics comes from two Papyri, namely the Rhine Mathematical Papyrus and the Moscow Mathematical Papyrus, which contain a collection of mathematical problems with their solutions. The Rhine Mathematical Papyrus, named after the Scottish man, A. H. Rhind, who bought it at Luxor in 1858. Moscow Mathematical Papyprus, bought by V.S. in 1893. Golenishchev was later sold to the Moscow Fine Arts Museum. The former papyrus was copied around 1650 by the scribe Ahmes from an original dating back to 200 years. The latter papyrus is approximately of the same period. The Rhine Papyrus has 85 problems and the Moscow Papyrus has 25 problems. In the first part of the papyrus, Ahmes deals with fractions of a unit (in modern) terms) it will be of the form Ahmes deals with the fractions of the unit, (in modern terms) they will be of the form. $2/2n-1$ where n stands for all odd numbers from 5 to 49.

For example, 2/29 can be written a $\frac{1}{24}+\frac{1}{58}+\frac{1}{174}+\frac{1}{132}$ See how close to the scribe was 24 58 174 132? Ahmes only records the result. It is possible that they have been worked out by former mathematicians, each of whom has worked in his own way or through repeated trials. It's hard to know how these results have been achieved.

Most of the mathematical works of the ancient times are concerned with the solution of problems to which various mathematical techniques are applied. Our study of these problems begins with several methods of solving linear equations. Many of the works take the solution of single linear equations for granted. Whenever such an equation appears to be part of more complex problems, the answer is merely given without any method of solution being mentioned. But these equations are explicitly dealt with by the Egyptian papyri.

The Rhine Papyrus and other Egyptian scrolls are mainly concerned with first-degree equations. Rhind Papyrus has a number of similar problems; all solved using the False Position method. The scribe's procedure can be considered as an algorithm for the solution of a linear equation. However, there is no discussion as to how the algorithm has arrived or how it works. It is clear, however, that the Egyptian scribes understood the basic idea of a linear relationship between two quantities, and that a multiplicative change in the first quantity implies the same multiplicative change in the second quantity.

Resnikoff and Wells have adopted a translation of Problem 28 of Rhind Papyrus from Eric Peet's book ‘hind Mathematical Papyrus' which is given below:

"Two-thirds added and one-third [of the sum] taken away, 10 remains. Make one-tenth of this 10: the result is 1, the remaining. Two-thirds of it, that is, 6 is added to it; a total of 15. A third of it is 5. It was 5 that were taken away: the remaining."

Non-linear polynomial systems

Current known techniques for the resolution of non-linear polynomial systems can be classified into symbolic, numerical and geometric methods. Symbolic methods based on the resultant and Gr}obner bases algorithms can be used to eliminate variables and thus reduce the problem to the end of univariate polynomial roots. These methods are rooted in algebraic geometry. However, current algorithms and implementations are ancient for low-grade polynomial systems consisting of up to three to four polynomials only. The ma jor problem arises from the fact that the computing roots of univariate polynomials may be ill-adjusted for polynomials of a degree greater than 14 or 15, as shown by Wilkinson. As a result, it is difficult to implement these algebraic methods using night precision arithmetic and slows down the resulting algorithm. As far as the use of algebraic methods in geometric and solid modelling is concerned, the current view is that they have led to a better theoretical understanding of the problems, but their practical impact is not clear.

The numerical methods for solving polynomial equations can be classified into iterative and homotope methods. Iterative techniques, like Newton's method, are only good for local analysis and work well if we give each solution a good initial guess. This is rather difficult for applications such as intersections or geometric constraint systems. Homotopy methods based on follow-up techniques have a good theoretical background and follow paths in a complex space. In theory, each path converges towards a geometrically isolated solution. They have been implemented and applied to a wide range of applications. In practise, there are many problems with the current implementations. The De

The erent paths being followed may not be geometrically isolated, causing problems with the robustness of the approach. In addition, continuous methods are considered to be computationally very demanding and currently limited to the solution of dense polynomial systems only. Recently, methods based on arithmetic interval have received a great deal of attention in computer graphics and geometric modelling. The resulting algorithms are robust, although their convergence may be relatively slow.

For some specific applications, algorithms have been developed using the geometric formulation of the problem. This includes subdivision-based algorithms for curves and surface intersection, ray tracing. In general, subdivision algorithms have limited applications and their convergence is slow. The Bezier Clipping improved their convergence. However, algebraic methods have been found to be the fastest in practise for low degree curve intersections. Similarly, algorithms based on the geometric properties of the mechanisms have been developed to address the problems of kinematics, constraint systems and motion planning.

Numerical Linear Algebra

The second half of the 20th century witnessed the maturation of the numerical linear algebra, which has become a well-established field of research. A multitude of reliable linear algebra numerical tools is well understood and developed. We will outline an independent value-based solution method in which linear algebra notions such as row and column space, zero space, linear (in) dependence and matrix rank play an important role, as well as essential linear algebra-based numerical tools such as singular value decomposition and self-value decomposition. The central object of the proposed method is the Macaulay matrix a sylvester-like structured matrix based on the coefficients of the multivariate polynomial set.

System Theory

We will borrow elements from system theory in general and from the theory of realization in particular. Realization theory will enter the scene when we analyse the null space structure of the Macaulay matrix. The link between the theory of realisation and the polynomial systems should not come as a surprise. For one-dimensional LTI systems, it is well known that the Sylvester matrix of the transfer function polynomial denominator is the left annihalator of the LTI system observation matrix. The theory of realisation of multidimensional (nD) systems has been linked to polynomial system resolution in using the Gr obner base view. In multidimensional dynamic systems, dynamics do not depend on a single independent variable (such as discrete time in linear time-invariant systems modelled by difference equations), but depend on several independent variables (such as space and time in PDEs). We will show that the null space of the Macaulay matrix can also be modelled as the output of an autonomous multidimensional (nD) system, possibly a singular (i.e. a descriptor) system. The null space of the Macaulay matrix has a multiplication structure that stems from the monomial structure of the unknown. By combining the structure of multiplication in the null space with the interpretation of the theory of realization, we develop an algorithm to create a (generalized) problem of self-value that delivers all the roots of the system.

Babylon

The major periods from which Babylonian mathematical texts exist are the Old Babylonian Age, between 1800-1600 BC and the Seleucid Age (200 BC). The old Babylonian era is contemporary with the Egyptian scribe Ahmes. Texts fall into one of the two categories, table texts and problematic texts. The text of the table consists of several parallel columns of numbers.

Problem texts provide various algebraic and geometric problems that need to be addressed. Table texts made around 1350BC which are available in the temple library of Nippur (in a reproduced form).

One of the reasons for the algebra development around 2000 BC is the use of the old Sumerian script by the new Semitic rulers. Like hieroglyphs, the ancient script was a collection of ideograms with each sign denoting a single concept. The signs expressed concepts, but were pronounced in a different way. Such ideograms were well suited for the algebraic language as our current ideograms'+,'‘-' etc. In Babylon's School of Administrators, this algebraic language has been part of the curriculum for many generations.

Linear algebra

Linear Algebra is a branch of mathematics designed to establish linear equation frameworks with a limited number of questions. Specifically, one might want to receive answers to the following questions:

Characterization of solutions: are there solutions for a given arrangement of linear equations? What is the number of solutions?

Finding solutions: How does the arrangement look like? What are the solutions to this? Linear Algebra is a systematic theory of linear equation framework solutions.

Model 1.2.1.1.1 Take the following two linear equations in the two questions x1 and x2:

$$\left.\begin{aligned} 2x_1 + x_2 &= 0 \\ x_1 - x_2 &= 1 \end{aligned}\right\}.$$

This system has a unique solution for x1, x2 ∈ R, namely

$$x_1 = \tfrac{1}{3} \text{ and } x_2 = -\tfrac{2}{3}.$$

The solution can be found in a few different ways. One approach is to first address one of the unknowns in one of the equations and then replace the outcome with the other equation. Here, for example, we might be able to solve to obtain $x_1 = 1 + x_2$ the second equation. At that point, replacing this in the first equation instead of x1, we've got to do it. $2(1 + x_2) + x_2 = 0.$

Out of this, x2 = −2/3. Then, by additional substitution,

$$x_1 = 1 + \left(-\frac{2}{3}\right) = \frac{1}{3}.$$

On the other hand, we can adopt a progressively systematic strategy in eliminating variables. Here, for instance, we can take away multiple times the second equation from the first equation so as to acquire

3x2 = −2. It is then prompt than $x_2 = -\frac{2}{3}$ and, by substituting this value for x2 in the first equation, that $x_1 = \frac{1}{3}$.

REVIEW OF LITERATURE

The word Diophantine suggests the Hellenistic mathematician of the third century, Diophantus of Alexandria, who made an investigation of such conditions and was one of the initial mathematicians to bring imagery into variable based math. The numerical investigation of Diophantine issues Diophantus began is right now called "Diophantine assessment." A straight Diophantine condition is a condition between two aggregates of monomials of degree zero or one.While individual conditions present a sort of astound and have been considered from the earliest starting point of time, the definition of general theories of Diophantine conditions was an achievement of the 20th century.

Adem Duru (2010) has thought about the exploratory showing technique (ETM) with the educator focused conventional instructing strategy dependent on students" achievement. This examination is led with 54 understudies, haphazardly separated into two gatherings; an exploratory gathering and a benchmark group. Test instructing strategy was utilized for the examination gathering and customary encouraging technique was utilized for control gathering. The test was applied to the two gatherings in two unique occasions. The principal test was applied previously and the subsequent test was applied after the educating. „t" esteem was utilized to analyze the two gatherings and the degree of essentialness was estimated as p According to the examination results, it was discovered that exploratory instructing technique was more powerful than instructor focused customary training strategy in the information and perception level.

Aryabhata(2013) portrays the estimation in just two sections of Aryabhatiya. His baffling segments were clarified by Bhaskara I sixteenth century in his talk AryabhatiyaBhasya. Bhaskara I appeared, Aryabhata's norm with a couple of models including 24 strong issues from stargazing. Without the explanation of Bhaskara I, it would have been difficult to decode Aryabhata's abstains. Bhaskara I suitably called the strategy kuttaka 4 (beating). The idea in kuttaka was later considered so noteworthy by the Indians that from the start the whole subject of polynomial math used to be called kuttaka-ganita, or just kuttaka.

Brahmagupta (2012) taken consideration of progressively problematic Diophantine conditions - he explored Pell's condition, and in his Samasabhavana he spread out a strategy to disentangle Diophantine conditions of the subsequent solicitation, for instance, 61x2 + 1 = y2. These procedures were dark in the west, and this very condition was acted like an issue in 1657 by the French mathematician Pierre de Format in any case, its answer was found only seventy years afterward by Euler. Meanwhile, various several years back, the answer for this condition was recorded by Bhaskara II (1150) using an adjusted type of Brahmagupta's strategy.

Carl Friedrich Gauss (2017) alluded to arithmetic as "the Queen of the Sciences". In the principal Latin Regina Scientiarum, similarly as in German Konigin der Wissenschaften, the word identifying with science infers (field of) data. Actually, this is in like manner the primary noteworthiness in English, and there is no vulnerability that arithmetic is in this sense a science. The specialization restricting the importance to typical science is of later date. If one accepts science to be cautiously about the physical world, by then arithmetic, or if nothing else unadulterated math, isn't a science.

Albert Einstein communicated that "to the degree the laws of arithmetic insinuate reality, they don't know; and asfar as they are certain, they don't suggest reality."

G. H. Hardy (2015) in A Mathematician's Apology conveyed the conviction that these smart thoughts are, in them, sufficient to legitimize the investigation of unadulterated arithmetic. Mathematicians normally attempt to find evidences of hypotheses that are particularly stunning, an excursion Paul Erdos much of the time insinuated as finding confirmations from "The Book" where God had recorded his favored verifications. The 12 pervasiveness of recreational arithmetic is another sign of the delight many find in comprehending numerical inquiries.

GottlobFrege (2016) was the originator of justification ism. In his crucial Die Grundgesetze der Arithmetik (Basic Laws of Arithmetic) he created number shuffling from an arrangement of justification with a general norm of gratefulness, which he called "Basic Law V" (for thoughts F and G, the development of F ascends to the growth of G if and only if for all articles a, Fa if and just if Ga), a standard that he took to be good as a significant part of method of reasoning.

Putnam(2015) At the moment that hypothesis discovers some sort of issue with science, a portion of the time science must be changed Russell's Catch 22 rings a bell, as does Berkeley's attack on the genuine moment anyway more consistently it is thinking that must be changed. I don't envision that the difficulties that perspective finds with old style science today are credible difficulties; and I feel that the philosophical translations of arithmetic that we are being offered on each hand aren't right, and that "philosophical understanding" is actually what arithmetic needn't mess with.

OBJECTIVE OF THE STUDY

1. To study the theory of solutions and the structure of solutions for linear algebra equations.
2. The study of polynomial equations appears in a number of mathematical models.

RESEARCH METHODOLOGY

Ancient and modern polynomial equations

Fourteenth century

In the fourteenth century, many changes in the European economy began to take place. It has had an effect on mathematics. The general cultural movement for the next two centuries is known as the Renaissance. It has had an impact in Italy in particular.

The Italian traders needed to be able to deal with the new economic circumstances. But the mathematics they needed had not been studied in the universities. They needed new tools to calculate and solve problems. To fulfill this requirement, a new class of'professional' mathematicians, maestri d'abbaco or abacists appeared in Italy at the beginning of the fourteenth century. Abacists wrote the texts from which they taught the necessary mathematics to the merchants' sons in the new schools created for this purpose. Trade revolutions soon spread to other parts of Europe as well.

All the algebraists of that period based their work on Islamic algebras, first translated into Latin in the twelfth century. But by the middle of the sixteenth century, all the works of Greek mathematicians, newly translated from Greek manuscripts in Constantinople into Latin, had been made available to European mathematicians.

The fourteenth-century Italian Abacists were instrumental in teaching traders the'new' Hindu-Arab decimal place-value system and algorithms to use it. The advantages of the new system made it possible to overcome initial hesitation. The old counting board system required not only a board, but also a pack of counters to be carried around. But only pen and paper were required for the new system. It could have been used anywhere. Abacists have taught new methods of calculation to entire generations of middle-class Italian children. These methods soon spread across the continent.

In addition to the Hindu-Arabic number system algorithms, abacists have taught their students methods of problem-solving using both arithmetic and Islamic algebra tools. Abacist texts are generally large compilations of problems along with their solutions. The abacus texts were designed not only for classroom use, but also to serve as reference manuals for traders themselves. The solution of a particular type of problem could easily be found and readily followed without the need for the merchant to understand the theory behind the solution.

Fifteenth century

Mathematical activity in the fifteenth century was concentrated mainly in the Italian cities and central European cities of Nuremberg, Vienna and Prague. Nicholas Cusa (1401-1464) is mainly remembered for his work on calendar reform and his attempts to square the circle and trise the general angle. Another mathematician, Georg von Peurbach (1423-1461), wrote arithmetic and works on astronomy. He has compiled a table of sins. Johann Muller (1436-1476) was the most influential mathematician of the century. It was generally known as the Regiomontanus. It was translated from the Greek works of Appollonius, Heron and Archimedes. Regiomontanus was invited to Rome by Pope Sixtus IV to take part in the reformation of the calendar. In his work De Triangulis Omnimodis, he used rhetorical algebra to solve geometric problems. This is the same as the fmding of an unknown part of the figure as the root of a quadratic equation.

According to Nicolas Chuquet recognised both positive and negative integral exponents. Luca Pacioli published Summa de Arithmetica, Geometrica, Proportioni and Proportionalita in 1494. It is briefly referred to as SfIma. This work has been compiled from a number of sources. It is a summary of the time's arithmetic, algebra and geometry. As per the False Position Rule is discussed and applied. The algebra in Suma was treated with quadratic equations. The algebra is syncopated by the use of abbreviations as p (from piu "more") to plus, m (from meno, "less") to minus. For equality, ae (from aequalis) is sometimes used.

DATA ANALYSIS

Polynomial equation and concept of algebraic equation

In the eighteenth century, the development of analysis and its applications in different areas formed the central aspect of the history of mathematics. In other areas, there was also important work. Throughout the century, several major texts were published in algebra, including the works of Maclaurin and Euler. Such books include some systematisation of earlier content. The text by Maclaurin contained a new method of solving linear equation systems. They are generally referred to as the Cramer Rule. In number theory, Euler's book contained some details on different methods. The main objective of algebra, however, was to extend the equation solving techniques of Cardano (1501-1576) and Ferrari (1522-1565) to fifth-degree and higher polynomial equations. In this effort, nobody succeeded. Lagrange produced a detailed review of the methods for solving cubic and quadratic equations towards the end of the century. He developed ideas that would be essential in dealing with equations of higher degrees.

Issac Newton (1642-1727)

Issac Newton was born in 1642, the year of Galileo's death, on Christmas Day. The four Newtonian books best known today are the Opticks and the Arithmetica universalis Principia, the'Method of Fluxions.' Between 1673 and 1683, the last work was composed, and it was first published in 1707. For the sums of the powers of the roots of a polynomial equation, this treatise includes the formulas, usually known as'Newton's Identities.'

In Arithmetica universalis, there is another theorem generalising'Descartes Rule of Signs' to determine the number of imaginary roots of Make a series of fractions whose denominators are 1,2, 3...up to the highest power in the equation, and whose numerators are the same numbers in the reverse order. Divide the second by the first, and the third by the second, and so on.

Approximate solution of equations

Newton has introduced a method for approximating the values of the real roots of the numerical equation. This applies equally to either an algebraic or a transcendental equation. This method is modified and is known as the Newton method, which is given as follows in:

"If f(x) = 0 has only one root in the interval [a, b] and if neither j'(x) nor f'(X) disappears in this interval, and if Xo is chosen as one of the two numbers a and b for which f(xo) and f(xo) are chosen. f"(XO) have the same sign, then $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ is nearer to the root than is x_0."

To construct a series for functions defamed implicitly, such as $y^3 + a^2y - 2a^3 + axy - x^3 = 0$.

Newton has used the method of successive approximations. He invented it. It's called the reversion method. This equation was called a 'affected equation'

CONCLUSION

They understood that a mathematical problem could not be considered resolved unless it could be shown that the solution is valid. They recognized the application of the axioms to the conversion of terms and the reduction of the implicit function of x to an explicit function. He used arithmetic techniques for unknown quantities. This was an early step in the arithmetic of algebra. Two important developments in the twentieth century on the specialization of mathematics as pure and applied strands. Topology is the most unifying branch of modem mathematics. The other main branches of pure mathematics are Abstract Algebra, Functional Analysis and Graph Theory. Algebra has been associated with most of the branches of modern mathematics. The growth of the algebraic structure was a powerful influence in the twentieth century. Galois has launched Group Theory for Equations. Galois wrote a summary of his findings in the theory of equations. Group Theory is the unifying power of the different branches of mathematics. During transformations, certain aspects of the algebraic equations of the geometric figure remain unchanged. the above account shows that the history of mathematics is a developing movement rather than a catalogue of achievements. The study shows that polynomial equations have evolved through a number of inspirations, errors, side-tracks and practical needs, not always rationally. Progress in this area has been made by passing through crises and by disproving or discarding arguments accepted as valid in the earlier period. It is hoped that this study would be useful for one to learn a lot about the evolution of concepts of polynomial equations with special reference in linear and quadratic equations, as well as to form generalized notions of unity and mathematical status.

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